Multiplying Permutations

$$a = (1, 3, 5, 2)$$

is a permutation.
What does this mean? It says 1 goes to 3, 3 goes to 5, 5 goes 2 , 2 goes to 1, and 4 and any other number is fixed. So we could write it like this.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 4 & 2 \end{pmatrix}$$

$$b = (1, 6, 3, 4)$$

is another permutation.
This says 1 goes to 6, 6 goes to 3, 3 goes 4 , 4 goes to 1, and 2, 5 and any other number is fixed. So we could write it like this.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 4 & 1 & 5 & 3 \end{pmatrix}$$

Next we look at multiplying these matrices. We will find ab. To do this we will start with the b permutation and then follow with a. (In some books you may see this done in the reverse direction, a first then b. There are different approaches to multiplying permutations here we will describe two of them.

$$ab = (1, 3, 5, 2)(1, 6, 3, 4)$$

So we begin with b, 1 goes to 6 where does 6 go to in a, 6 is fixed so 6 goes to 6 so now we know our first entry is 1 goes to 6.
Next back to b where does 6 go in b, 6 goes to 3, where does 3 go to in a, 3 goes to 5, Now we have our next entry 6 goes to 5.
Next back to b where does 3 go in b, 3 goes to 4, where does 4 go to in a, 4 is fixed in a therefore 3 goes to 4, Now we have our next entry 3 goes to 4.
Next back to b where does 4 go in b, 4 goes to 1, where does 1 go to in a, 1goes to 3, Now we have our next entry 4 goes to 3.
Next back to b where does 2 go in b, 2 is fixed so 2 goes to 2, where does 2 go to in a, 2 goes to 1, Now we have our next entry 2 goes to 1.
Now for our last entry we go back to b where does 5 go in b, 5 is fixed in b so 5 goes to 5, where does 5 go to in a, 5 goes to 2, Now we have our next entry 5 goes to 2.
So our permutation looks like this

$$\begin{pmatrix} 1 & 6 & 3 & 4 & 2 & 5 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$$

So our final ab is

$$ab = (1, 6, 5, 2)(3, 4)$$

Or we could use this method - We write out b as in number 4 above. Then use the second line in b to find where these values go in a and fill these results into a third line. We then cross out the middle line and we have our resultant permutation ab =

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 4 & 1 & 5 & 3 \\ 6 & 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

So eliminating the middle line we get ab as in 6 and 7 above

$$ab = (1, 6, 5, 2)(3, 4)$$

Examples to try yourself. In each case find ab

1.

$$a = (1, 5, 2, 4)$$
$$b = (2, 6, 5)(3, 4, 7)$$

2.

$$a = (1, 2, 5, 3, 4, 6)$$
$$b = (1, 5, 3, 7, 4)$$

3.

$$a = (1, 4, 6, 3, 7)(2, 8)$$
$$b = (2, 5, 3)(4, 7, 8, 1)$$

Answers:

1. **(1,5,4,7,3)(2,6)**
2. **(1,3,7,6)(2,5,4)**
3. **(1,6,3,8,4)(2,5,7)**

Note: Often the commas between the elements of the permutation are removed i.e. (1,5,4,7,3)(2,6) = (1 5 4 7 3)(2 6)